Aristotle, Archimedes, Euclid, and the Origin of Mechanics: The Perspective of Historical Epistemology

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The Law of the Lever from the Perspective of a Long-Range History of Mechanical Knowledge

The science of mechanics has a history extending over more than two millennia. Its origin is closely associated with the name of Archimedes and his proof of the law of the lever. But the law of the lever was not only among first mechanical laws to be formulated and proven, it also played a dominant role throughout the history of mechanical knowledge. On the basis of work undertaken at the Max Planck Institute for the History of Science in cooperation with a number of colleagues,[1] we will in the following analyse the origin of mechanics from the perspective of a "historical epistemology" as we pursue it at the Institute. Historical epistemology in this sense aims at understanding the structures of such long-term developments of knowledge.

Periods of mechanical knowledge

A rough survey suggests that the long-term history of mechanical knowledge can be divided into six more or less coherent periods:

The first period may simply be called the "prehistory of mechanics"; it comprises the long period of time in which human cultures accumulated practical mechanical knowledge without documenting this knowledge in written form and without developing theories about it. Although the origin of other sciences such as mathematics and astronomy can be traced back to the ancient urban civilizations of Babylonia and Egypt, this, surprisingly, is not the case for mechanics. In fact, although there are numerous sources testifying to the large construction projects of these civilizations, there is no single document referring to the mechanical knowledge that must have been involved in these endeavours.

The next period is that which properly merits the label "origin of mechanics." It saw, in particular, the formulation and proof of the law of the lever. More generally, it is characterized by the appearance of the first written treatises dedicated to mechanics and to physics, associated in particular with names such as Aristotle, Euclid, Archimedes, and Heron. These works had an enormous impact on subsequent development. Aristotelian physics, in particular, provided the conceptual basis for physical theories until the advent of classical mechanics.

The third period is, in its beginning, characterized by the transformation of mechanics into a "science of balances and weights" in which the law of the lever again played a key role. This period covers the Arab and Latin Middle Ages, which saw the production of an extensive mechanical literature focused, however, on a relatively small range of subjects.

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1 Markus Asper, Istvan Bodnar, Brian Fuchs, Elke Kazemi, and Paul Weinig.

The fourth period is that of preclassical mechanics, ranging from the sketches of Renaissance engineers such as Leonardo da Vinci to the mature works of Galileo Galilei. In contrast to the preceding period it deals with an increasingly large number of subjects, among them the inclined plane, the pendulum, the stability of matter, the spring, etc. Nevertheless, the law of the lever continued to play an important role also for the foundation of preclassical mechanics.

The fifth period is that of the "rise of a mechanistic world view." It extends from the first comprehensive visions of a mechanical cosmos such as that of Descartes, via the establishment of classical and later analytical mechanics, to the attempts of 19th century scientists to build physics on an entirely mechanical basis.

The sixth period comprises the decline of the mechanical world view and the disintegration of mechanics at the turn of the 19th to the 20th century and is associated with the emergence of modern physics and its conceptual revolutions represented by the relativity and quantum theories.

An overview of the long-term development of mechanics raises a number of puzzling questions. For example: How did (theoretical) mechanics originate in ancient Greece and why did this not happen earlier? What kind of knowledge made the formulation of the law of the lever possible, and what knowledge was required for its proof? What accounts for the remarkable differences between the medieval science of weights and preclassical mechanics? What kind of empirical knowledge made the emergence of classical mechanics possible and what accounts for its remarkable stability over the more than 200 years of classical physics? What explains the even greater stability of Aristotelian physics over more than 2000 years? How can one explain the disintegration of mechanical concepts around the turn of the last century and how could the emergence of revolutionary theories such as the theory of general relativity, which proved to be the foundation for knowledge, actually not be available at the time of their creation? And how did the law of the lever survive all these changes?

Three types of knowledge

In view of the remarkable continuities and discontinuities of the development of mechanical knowledge it may be tempting to look for contingent reasons which shaped its history, and which are unrelated to the intrinsic nature of mechanical knowledge. But is it really plausible to explain, for instance, the long dominance of Aristotelian physics, which even extended up to the period of preclassical mechanics with its widespread anti-Aristotelian attitude, merely by external factors such as the adoption of Aristotelian philosophy as the official doctrine of the Catholic Church? Such explanations only sound convincing if one assumes that scientific knowledge is exclusively represented by scientific ideas and theories. However, if one takes other dimensions of knowledge into account such as the intuitive knowledge governing thinking and behaviour in our natural environment, it becomes rather more plausible to assume that certain aspects of Aristotelian physics were as convincing for medieval and early modern scholars as they are for children and even high-school students today.

In short, we would like to suggest that an understanding of the long-term development of mechanical knowledge must take into account, in addition to the theoretical knowledge usually considered in the history of science, two further types of knowledge, intuitive physics and practical mechanical knowledge.

Intuitive physics is based on experiences acquired almost universally in any culture by human activities. Experiences relevant to intuitive mechanical knowledge include, for instance, the perception of material bodies and their relative permanence, their impenetrability, their mechanical qualities, and their physical behaviour. Intuitive physics not only forms the basis of practical human activities but also of the arguments of scientific theories of mechanics. In proofs of the law of the lever, it is, for instance, usually assumed tacitly and without any need of justification that if one arm of the balance goes up, the other one cannot go up as well but necessarily must go down.

A second kind of mechanical knowledge which predates any systematic theoretical treatment of mechanics is practical mechanical knowledge—the knowledge achieved by dealing with mechanical tools such as the balance. In contrast to intuitive mechanical knowledge, this type of knowledge is no longer universally shared by every human being. It is closely linked to the production and use of such tools by professionalized groups of people, and it consequently develops in history.

Mental models

An analysis of the relation between the various layers of knowledge and their development—the aim of an historical epistemology—requires an appropriate description of the architecture of knowledge. In our approach to historical epistemology, we make use of the concept of "mental models," taken over from cognitive science and adapted to the needs of an historical epistemology.[2] Mental models are knowledge representation structures which allow for drawing inferences from prior experiences about complex objects and processes even when only incomplete information on them is available. Furthermore, conclusions based on mental models can be corrected in the light of new information, in contrast to monotonic deductive systems, in which a valid inference is not affected by the addition of new premises.

A mental model consists of a relatively stable network of possible inferences relating inputs that are variable. Cognitive science often uses the term *slots* to indicate the nodes in the structure which have to be filled with inputs satisfying specific constraints. Applying a mental model presupposes the assimilation of specific knowledge to its structure, that is, input information compatible with the constraints of the slots is mapped into them. Filling the slots is the crucial process that decides on the appropriateness and applicability of a mental model for a specific object or process. Once the mapping is successful—if the input information satisfies the constraints of the slots—the reasoning about the object or process is, to a large extent, determined by the mental model.

Let us consider the example of the "motion-implies-force" model for instance, which, when involved in the interpretation of a process of motion, yields the conclusion that the moved object is moved by a force exerted upon it by some mover. While this conclusion is incorrect from the perspective of classical physics, contradicting as it does Newton's principle of inertia, it is in agreement with Aristotelian dynamics. What is more important in our context, the "motion-implies-force" model represents elementary human experiences. In fact, when observing some moving object, for instance a car moving on the road, one usually presumes that there is some

[2] See (Gentner and Stevens 1983; Renn 2000).

mover at work which drives the object by its force, even when the mover itself and its force cannot be directly observed. The missing information about the mover is simply added by the default settings of the model based on prior experiences. If, however, additional empirical information eventually becomes available, such as when a closer look reveals that the car is not actually being driven by its engine but rather pushed by its driver, then this information replaces the original default settings without, however, challenging the model itself.

Mental models relevant to the history of mechanics either belong to generally shared knowledge or to the shared knowledge of specific groups. Accordingly, they can be related to the three types of knowledge introduced earlier. First, there are the basic models of intuitive physics, such as the motion-implies-force model just described. Another group of mental models is part of the professional knowledge of more or less specialized practitioners. Their historical transmission is related to the transmission of the real instruments that embody them. And, finally, there are the mental models which belong to theoretical knowledge and which are communicated by an explicit description of their structure and of the conditions of their applications.

Again, let us consider examples. A foundational experience of practitioners' knowledge since ancient times has been the equivalence of the weight of a body and the force required to lift it up. This equivalence is prototypically embodied in a real model, namely that of the balance with equal arms. In fact, the force which keeps the balance in equilibrium is equal to the weight in the scale pan. We hence call this model of compensation between force and weight the "equilibrium model." However, the practical knowledge of the technicians and engineers of Antiquity also involved other basic experiences, and, in particular, the experience of how one can free oneself from the constraint of the equivalence between weight and force. In fact, the art of the mechanician consisted precisely in overcoming the natural course of things with the help of instruments such as the lever. According to this understanding, a mechanical instrument serves to achieve, with a given force, an "unnatural" effect that could not have been achieved without the instrument. We have therefore called the model underlying this understanding the "mechanae model"—according to the Greek word "mechanae" which means both mechanical instrument and trick, and which is at the origin of the word *mechanics*.

After this survey of the epistemological framework of our analysis, let us turn to the proofs of the law of the lever by Archimedes and Euclid in order to analyse their common epistemic roots.

The Aristotelian Origin of Mechanics (I)

The origin of the law of the lever

The first encounter between theoretical and practical knowledge had long since taken place before the texts of Archimedes and Euclid that are usually considered the classical references for the discovery of the law of the lever were written. This encounter is represented by the first surviving treatise on mechanics, the so-called "Mechanical Problems" traditionally and, as we believe, correctly ascribed to Aristotle, who was born in 384 B.C. about a century before Archimedes. This treatise is centred around the question:[3]

[3] Aristotle, *Mechanical Problems,* 850a30.

> Why is it that small forces can move great weights by means of a lever?

The answer is given on the basis of a principle which is repeatedly applied in the treatise:[4]

> Moved by the same force, that part of the radius of a circle which is farthest from the centre moves quicker than the smaller radius which is close to the centre.

While this principle does not express the law of the lever as we are accustomed to it, it nevertheless comes so close to its formulation that we may consider it as its direct precursor. In fact, according to this principle, the same force can compensate an ever greater weight, the further it is away from the centre or fulcrum of a lever. The law of the lever now merely specifies that the weight that can be compensated in this way is proportional to the distance of the force from the centre. According to our analysis of Aristotle's text, the knowledge structures it displays emerged from a reflection of experiences made possible by the invention of the balance with unequal arms, an invention that had taken place only recently.[5] These knowledge structures are determined by a specific mental model resulting from an integration of the mechanae model with the equilibrium model, a model that we have called "the balance-lever model." This model can indeed be understood as a generalization of the equilibrium model associated with the ordinary balance with equal arms. In the case of an equal-arms balance, weight differences are balanced by weights; in the case of an unequal-arms balance, they are balanced by changing the position of the counterweight along the scale or, as in Aristotle's case, by fixing the counterweight at the end of the beam and changing the position of the suspension point. This necessarily generalized the equilibrium model: weights can be compensated not only by weights but also by distances. It was thus the practical knowledge related to the balance with unequal arms that provided the empirical basis for the formulation of the law of the lever. In a sense, the law of the lever is even stated at one point in Aristotle's treatise, when he sums up:[6]

> The weight moved is to the moving weight inversely as the length to the length.

This proposition, however, comes somewhat as an afterthought and is never taken up again in the entire treatise; thus it may well be that it actually represents the later insertion of a commentator, which entered the text when it was copied. While the question whether or not Aristotle himself actually formulated the law of the lever must therefore be left open, it is clear that the law must have been well known when it was given a proof a generation or two later by Euclid and Archimedes.

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4 Aristotle, *Mechanical Problems*, 848b4-6.
5 See Damerow, Renn, and Rieger 2002.
6 Aristotle, *Mechanical Problems*, 850b2.

Archimedes' Proof of the Law of the Lever

The key idea of Archimedes' proof

The treatise of Archimedes on the equilibrium of planes contains, at its beginning, a demonstration of the law of the lever, formulated in the sixth and the seventh proposition.[7] The treatise makes use of sophisticated mathematical arguments involving, for instance, the distinction between commensurable and incommensurable quantities. Nevertheless, the key idea of Archimedes' proof can be expressed in relatively simple terms by taking a specific example.

- Consider a (weightless) beam which is divided into 6 equidistant units and which is supported in its middle. Consider, furthermore, two weights, one composed of 4 units of weight, the other consisting of 2 units of weight.
- Now take the 6 units of weight and place each of them at the middle point of one of the 6 sections of the beam (figure 1). It is then immediately clear that the balance will be in equilibrium.
- Next assume that the effect of the 4 units of weights does not change when they are placed not one-by-one on the beam but when they are concentrated at their middle point as shown in figure 2. Similarly assume that also the effect of the 2 units of weight does not change if they are conceived as being concentrated at their middle point.

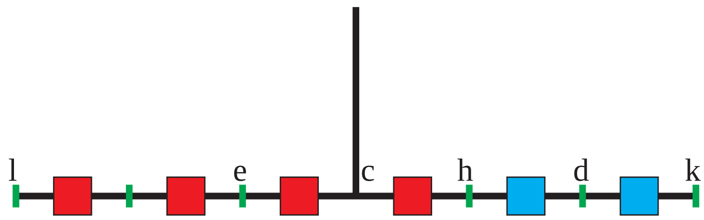

Figure 1

[7] Archimedes, *On the Equilibrium of Planes*; Heiberg 1910, 132-138, See (Clagett 1959, 34-37).

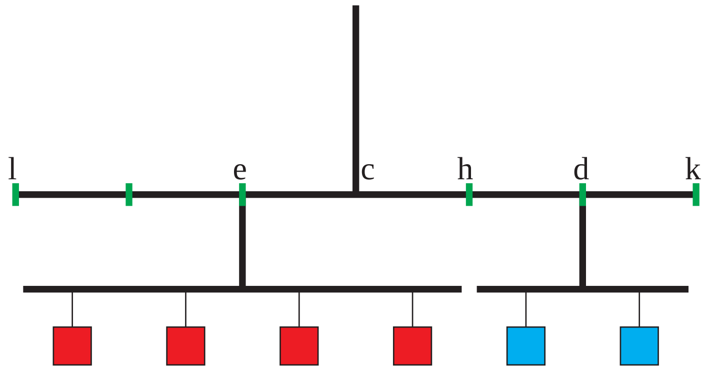

Figure 2

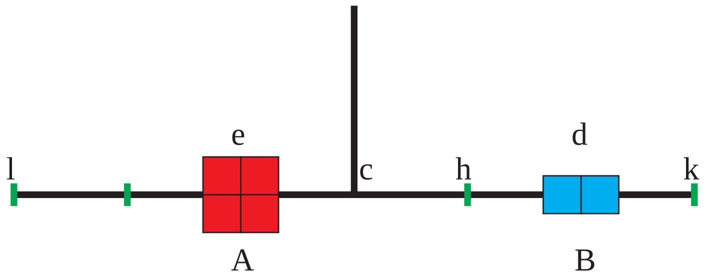

Figure 3

- In other words, the equilibrium of the total configuration remains unchanged when the original weights of 4, respectively 2, units are placed at these middle points (figure 3). We have thus arrived at a situation in which the weight A of 4 units is placed at a distance of 1 unit and the weight B of 2 units is placed at a distance of 2 units. If the equilibrium is not changed by this concentration, we have thus established a special case of the law of the lever.

The general proof essentially follows the same line of reasoning which is carefully prepared by a number of postulates stated in the beginning of the treatise and the preceding 5 propositions derived from them. As has already been indicated, the proof of the law of the lever is then performed separately for the case of commensurable and incommensurable quantities.

Mach's critique of Archimedes' proof

Clearly the critical point of Archimedes' proof is, however, not its mathematical part but the last step in our analysis, i.e. the question of the legitimacy of substituting a group of equal weights, placed at equal distances on a beam, by a single weight equal to their sum and placed at the middle point of the distance spanned by these weights.

The legitimacy of this argument has been often been disputed, in particular vividly and powerfully by the historian and philosopher of science Ernst Mach around the turn of the last century.[8] He argued that this step actually presupposes what has to be shown, the law of the lever. In fact, he argued, this step involves the assumption that equal displacements of a weight placed on a beam from and towards the point of support cancel each other, which assumes that the effect of a weight placed on a beam is a linear function of distance, a presupposition essentially equivalent to the law of the lever.

A closer look at Archimedes' proof reveals, however, that he does not actually talk about such displacements of weights at all. This objection to Mach's analysis has been raised by several historians and has been masterfully elaborated in Dijksterhuis's book on Archimedes.[9] In his analysis Dijksterhuis correctly emphasizes that, in the critical step of his proof, Archimedes makes use of the concept of centre of gravity in order to justify that the original weights keep the system in equilibrium. Indeed, Archimedes argues that these weights maintain the equilibrium because they are placed at the respective centres of gravity of the two groups of equally spaced weights which correspond to them and which, taken together, keep the beam in equilibrium because their overall centre of gravity coincides with the point of support of the beam.

Without analysing the course of Archimedes' line of argument in detail it is clear that his use of the concept of centre of gravity essentially presupposes three properties:

1. The centre of gravity of a symmetric configuration as used in the proof will be at the middle point of the configuration.
2. If a body is supported at (or suspended from) its centre of gravity, it will be in equilibrium.
3. Bodies of equal weight may be substituted for each other (whatever their suspension) without changing the state of equilibrium as long as their centres of gravity coincide.

As a matter of fact, these properties are all introduced in the earlier part of the treatise, either in the postulates or in the propositions that are demonstrated. In particular, the first property is explicitly demonstrated in the fifth proposition, the second property is introduced as an apparently self-evident property of the centre of gravity in the proof of the fourth proposition, and the third property is formulated, as it seems, somewhat obscurely as the sixth postulate which reads:[10]

> If magnitudes at certain distances be in equilibrium, other [magnitudes] equal to them will also be in equilibrium at the same distances.

The term "magnitudes" in the formulation of this postulate is indeed somewhat surprising, differing as it does from the use of the term "weight" elsewhere in the postulates. In the propositions this term occurs whenever they also involve the notion of the centre of gravity. And indeed, "magnitude" is used by Archimedes to denote a generic body of unspecified shape insofar as it can be represented by its centre of gravity both with regard to its weight and its position. The sixth postulate hence claims that bodies which are equal magnitudes in this abstract sense can also

8 Mach 1988, 10-24. Mach also reports other criticisms.
9 Dijksterhuis 1956, Chapter 9.
10 Archimedes, *On the Equilibrium of Planes*; Heiberg 1910, 124; Clagett 1959, 31. In the definitions of Book V of the *Elements* Euclid defines the term "magnitude" in terms of itself so that it is in fact only implicitly defined by its actual use in the arguments.

be substituted for each other if placed on the lever (or suspended from a balance)—without disturbing the state of equilibrium. With this understanding, the crucial step of Archimedes' proof is apparently justified.

On what knowledge is the proof based?

But is it really justified? Let us to return once more to Mach's criticism. The starting point of his analysis was amazement about the very possibility of Archimedes' proof:[11]

> From the mere presupposition of the equilibrium of equal weights in equal distances the inverse proportion between weight and lever arm is being derived! How is that possible?

The above analysis has indeed hardly refuted the legitimacy of Mach's quest for the epistemic foundation of Archimedes' proof. What is the knowledge on which this proof is based? This question is best answered with the help of our description of knowledge structures in terms of mental models. Archimedes' concept of magnitude, in connection with the concepts of weight and centre of gravity, indeed works like the mental models introduced above—we shall refer to the corresponding model as the "centre of gravity model." It can be applied to any heavy body, allowing us mentally to replace it by its total weight and its centre of gravity. Its slots are therefore the heavy body itself, its total weight, and the centre of gravity. The structure of the model is determined by noting that any axis through the centre of gravity turns the body into a lever in equilibrium, or, in the words of Pappus:[12]

> We say that the centre of gravity of any body is a point within that body which is such that, if the body be conceived to be suspended from that point, the weight carried thereby remains at rest and preserves its original position.

In other words, the centre of gravity model allows any body to be conceived as a generalized balance with a fulcrum and a distribution of weights around it in equilibrium. In contrast to the fulcrum, however, the centre of gravity no longer has to be a physically distinguished point that can be identified by visual cues but its identification is rather the result of the application of the model to a heavy body. In fact, the centre of gravity model can be applied to every body whether it physically resembles a balance or not. This is the step taken by Archimedes in his work on the equilibrium of plane figures.[13]

To what kind of knowledge does the centre of gravity model belong? It is clearly rooted in practical knowledge dealing with balances as it is embodied in the equilibrium model and also in observations on the stability of bodies. There are, on the other hand, indications of the existence of an earlier barycentric theory collected by Dijksterhuis, which are, however, based only on passages in later works by Heron and Pappus which still display the deficiencies in the notion of the centre of gravity that Dijksterhuis ascribes to this earlier theoretical tradition.[14] But also the text by Archimedes itself makes it sufficiently clear that understanding the centre of gravity model

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[11] Mach 1988, 14.
[12] Pappus, *Collections*; Hulsch 1965, 1030-31. See (Gerhard 1871, 310-11; Dijksterhuis 1956).
[13] Archimedes, *On the Equilibrium of Planes*, Heiberg 1910, 124-138. See (Clagett 1959, 31-37).
[14] Dijksterhuis 1956, 298-300.

actually requires an explicit or implicit description of its properties. In other words, neither the emergence nor the transmission of this mental model is conceivable without its representation by written language. The very fact that the model is applicable to all heavy bodies suggests that it could hardly have emerged in the context of practitioners' knowledge dealing with specialized domains but that the model rather belongs to theoretical knowledge.

It is therefore plausible to assume that the centre-of-gravity model resulted from a reflection on the applicability of the equilibrium model to all bodies. Indeed, the application of a mental model to different objects and processes and the outcome of such applications may become themselves the object of reasoning that produces new knowledge, provided that such knowledge is appropriately represented—in our case by written language. Knowledge about knowledge structures may then in turn change these knowledge structures. Thus, the application of a mental model may lead to changes—in our case to a generalization—of that model by a deliberate reorganization of its structure as the result of the accumulated meta-knowledge obtained by reflection.

As an example for such a reorganization take the transformation of the concept of fulcrum into that of the centre of gravity. While in the equilibrium model the fulcrum is primarily characterized by its physical properties as the turning point of a balance, and only then by the functions it takes on as a consequence of the application of the model, in the more developed model, these secondary properties now become the primary properties of the centre of gravity. Because of the new abstract quality which the concept of fulcrum assumes when generalized to the concept of centre of gravity, it can now be applied iteratively, making it possible, in particular, to conceive the point of suspension of a weight on a balance in turn as the new fulcrum of another balance. This iterative application of the concept of centre of gravity is in fact the crucial feature of Archimedes' proof. As figure 2 shows, even this iteration may still be visualized as a complex combination of balances—with the important difference, however, that the substitution operations necessary to make Archimedes' proof work are justified only for the abstract concept of centre of gravity and not for concrete balances.

Our answer to the question of the epistemic roots of Archimedes' proof of the law of the lever can hence be summarized as follows: The proof makes essential use of the centre-of-gravity model which results from a reflective abstraction of the equilibrium model rooted in practical knowledge, made possible because of the representation of this knowledge in terms of written language. Although the postulates with which Archimedes' work begins make no mention of levers, balances or fulcrums, but only speak of weights, magnitudes, distances, and centres-of-gravity, they nonetheless actually describe operations of adding and taking away weights on a balance.[15]

Euclid's Proof of the Law of the Lever

The key idea of the proof

Fortunately, the far-going implications of this interpretation of Archimedes' proof for an epistemological understanding of the origins of theoretical mechanics can be checked by comparing it with another early proof of the law of the lever ascribed to Euclid. Euclid's proof is preserved only

[15] Archimedes, *On the Equilibrium of Planes*, Heiberg 1910, 124, Clagett 1959, 31.

in Arabic and its ascription to Euclid is even somewhat doubtful.[16] The following discussion is restricted to a description of the proof's key idea and leaves out most of the technicalities, just as we have done for Archimedes' proof. Euclid's proof starts with a description of a peculiar set-up: Imagine a cross-like structure formed by rigid but weightless beams supported at its middle point (figure 4).[17] One corner of the cross is closed by two further weightless beams forming a square. Since all beams are weightless, the planar structure supported in the middle will obviously be stable. It will also remain in equilibrium if three equal weights are positioned on it, two at the open ends of the cross at *a* and *e*, and one at the corner point *h* of the square. The author justifies this fact by imagining that each of the main beams of the structure can be conceived as the arm of a balance, moveable only around an axis formed by the other beam. Then one of the weights is placed right on the axis of rotation and has hence no effect on the balance, while the other two weights are placed at equal distances from the fulcrum and hence keep the balance in equilibrium. The fact that one of these weights is directly attached to the beam while the other is attached at a certain distance from it by means of another of the beams forming the complex structure is considered to be of no effect since the latter beam comes off vertically from the beam of the balance so that it does not affect the distance of the weight from the axis of rotation.

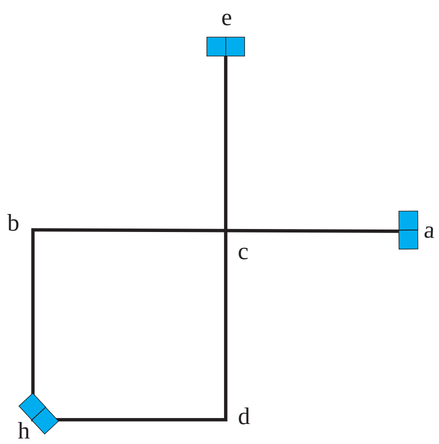

Figure 4

Since this structure is hence in equilibrium around any conceivable axis of rotation, one can imagine that one such axis lies along the diagonal of the square crossing the suspension point (figure 5). Without changing the distance from this axis of rotation, the two weights at the open ends of the cross can now be moved towards this axis until they form one weight *w* of two units lying on this axis. The distance *l* of this double weight *w* from the fulcrum *c* is now just one half the distance of the weight at the end point of the square *h* which has not changed its place. In other words, a special case of the law of the lever has been demonstrated.

[16] Euclid, *The Book of the Balance*, Clagett 1959, 24-30.
[17] Euclid actually starts with a real beam of uniform thickness rigidly fastened to an axis; then he abstracts from both aspects when giving the proof.

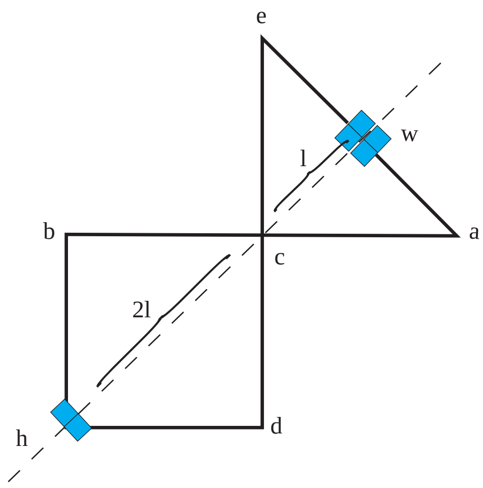

Figure 5

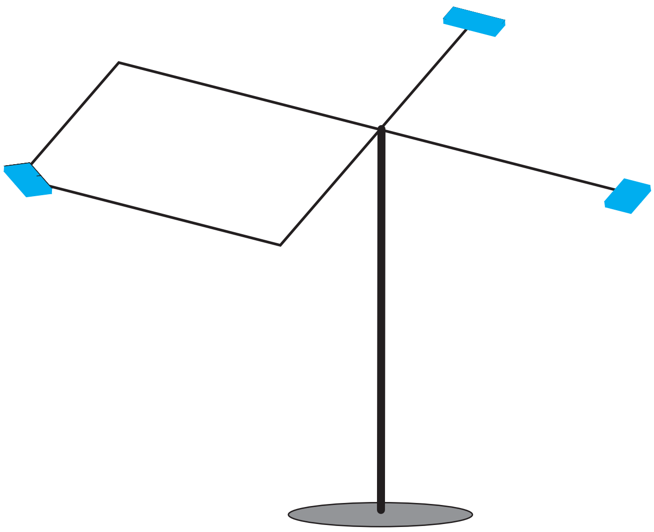

Figure 6

Euclid's proof and Mach's objection

The proof as we have so far considered it displays a number of remarkable features. It refers to a complex planar structure that at first sight has little in common with balances familiar from practical experience. What has been shown so far is merely a special case of the law of the lever. How can this special case be generalized? The sequel of the proof pursuing such a generalization appears to be even more complex. First of all, the planar construction is generalized from a square to rectangular figure. At first sight, little is achieved in this way. It is merely shown that one weight can be in balance with two other weights that are displaced by equal distances from their original positions at the fulcrum of the balance and at the end of the other lever arm, respectively. The crucial point, however, is that the insight into the legitimacy of such displacements is then taken as the basis for justifying an approach similar to that familiar from Archimedes' proof. In fact, if it is possible to move two equal weights by equal distances, one towards, the other away from the fulcrum, then it is possible to transform an equal distribution of weights along the beam of a balance into one weight concentrated at its middle point. In other words, the author so demonstrated the legitimacy of the operations criticized by Mach because they were supposedly used by Archimedes. In fact, however, they occur only in the proof of Euclid but here as the result of a complex justification based on projecting displacements in the plane onto displacements along the beam of a lever. In order to characterize this quality of weights, Euclid even introduces a term characterizing the effect of a weight on a balance, the force of heaviness.[18] In terms of this concept, his proof amounts to showing that the displacements leave the force of heaviness unchanged, just as Archimedes' proof amounts to showing that the centre of gravity is left unchanged by passing from a symmetric to an asymmetric constellation.

On what knowledge is the proof based?

The most remarkable aspect of Euclid's proof, if compared to that of Archimedes, is perhaps the fact that it proceeds without involving the concept of centre of gravity. On what knowledge then is Euclid's proof based? Like Archimedes' proof it starts from a number of assumptions that are closely related to the equilibrium model associated with the ordinary balance. But again, as it was the case of Archimedes' centre-of gravity model, these assumptions actually characterize a mental model far more general than the equilibrium model, a model one could call the "force-of-weight model." Similar to the centre-of-gravity model, it was based on practical experiences, in particular on the experience gained with unequal-arms balances that differences of weight can be compensated by differences of length. But also similar to the centre-of-gravity model, the theoretical character of this model depended on the formulation of its properties in terms of written language. As a matter of fact, Euclid's proof starts from rather artificial looking assumptions about the indifference of the equilibrium state with regard to displacements of the weights perpendicular to the axis of rotation. While these assumptions can easily be made plausible for the default case of an equal-armed balance, they determine a mental model with a much greater range of applicability, including, in particular, such strange planar constructions as used in the proof. As a matter of fact, the statements of the properties of the model are evidently formulated in such a way that they are apt to legitimize precisely the operations that need to be performed on these constructions in order to make the proof work. The postulates may hence be considered as the result of a reflection on such operations. But in spite of the contrived character of Euclid's postulates, the theoretical model he used had as wide a range of applicability as Archimedes' centre-of-gravity model. Both constitute reflective abstractions not just of the specific operations used in the proofs but generally of operations on balances.

Our answer to the question of the epistemic roots of Euclid's proof of the law of the lever can hence be summarized as follows: The proof makes essential use of the force-of-weight model which

[18] qūwat al-thiql. See Clagett 1959, 27.

results from a reflective abstraction of the balance-lever model rooted in practical knowledge, made possible because of the representation of this knowledge in terms of written language.

The Aristotelian Origin of Mechanics (II)

The above analysis of the proofs of Archimedes and Euclid has highlighted the role of written representations of practical knowledge as starting point for the theoretical reflection yielding the abstract concepts at the core of theoretical mechanics. This brings us back to Aristotle's "Mechanical Problems," which in fact constitutes a kind of missing link between the tradition of practical knowledge and the theoretical tradition usually considered to constitute the origin of mechanics as a science. Indeed, Aristotle's text not only transposes the experience of practitioners with unequal-armed balances into the medium of written language, formulating the balance-lever model which captures the practioners' experience that differences of weight can be compensated also by differences of lengths, thus providing the stepping stone for Euclid's proof based on the force-of-weight model. It also contains a first generalization of the equilibrium model to the case of a balance with a material beam, that is, a beam which itself possesses weight, thus offering the crucial stepping stone for Archimedes' proof based on the centre-of gravity model, as we shall now see. Considering an equal-arms balance with an extended, material beam, it is necessary to distinguish between the case in which the balance is suspended from above and the case in which it is supported from below. In fact, a balance displays different behaviours when its equilibrium is disturbed by adding or removing a weight in these two cases, as Aristotle's question makes clear:[19]

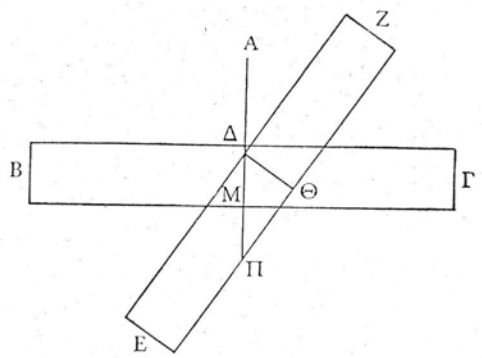

Figure 7. Why is it that, when one removes the weight that has inclined the balance downwards, it rises again, but that, when it is supported from below, the balance does not rise but remains where it is?

The answer to Aristotle's problem is based on considering the perpendicular line across the point of suspension which represents a plane dividing the balance in two parts. The relation between the weights of these two parts of the balance now decides whether or not the balance rises again. In this way, the equilibrium model is generalized to apply to the suspended beam itself, without the

[19] Aristotle, *Mechanical Problems*, 850a3-6.

weights usually attached to a balance. The criterion for whether it moves or remains at rest is now no longer the relation between such weights but that between the two parts divided by the perpendicular plane across the point of suspension. Although applied to the special case of the material beam of a balance either suspended from above or supported from below, this model works quite generally for all bodies and if pursued naturally singles out the case in which the two parts are always of equal weight. If one moves the suspension point down through the beam and moves the fulcrum up through the beam, one reaches a point where the downwardly displaced side of the beam is neither greater nor lesser than the other side. For the material beam this happens if it is suspended in the middle rather than from above or below, in other words, if it is suspended from its centre of gravity—a conclusion that Aristotle does not in fact draw although his explicit program is to identify suspension point, fulcrum, and centre of the circle. And he only compares the sizes of the areas without taking into account whether the area and thus the weight is evenly distributed on both sides of the suspension point.[20]

Aristotle's argument thus provides, when read in reverse, a first characterization of the centre of gravity as the point from which suspended a body will remain at rest and preserve its position. This characterization is exactly the definition of the centre of gravity later given by Pappus and ascribed to an early tradition of barycentric theory by Dijksterhuis. Our analysis suggests that this tradition goes actually back to the Aristotelian "Mechanical Problems" and hence to what probably represents the first encounter of theoretical tradition and practical traditions.

The Character of the Ancient Proofs

In conclusion, what is the character of the ancient proofs of the law of the lever from the perspective of historical epistemology? Where do the concepts on which these proofs depend come from and what makes these proofs convincing? The law of the lever, the centre of gravity, the indifference of the effect of a weight with regard to displacements perpendicular to the axis of rotation of a balance, the force of heaviness on a balance changing with its position, all of these concepts are, as we have seen, reflective abstractions resulting from mental models rooted in practical experience, in particular the equilibrium model and the balance-lever model. The representation of these models in the medium of written language constituted not only the basis for using these models far beyond the original extension of their range of applicability but also the presupposition for reflecting on the properties of the model as they are revealed by "running or applying it." Whereas the original mental model emerged from a reflection on the operations directly performed with the real object, secondary abstractions such as the centre of gravity or the force of a weight resulted from a reflection on mental operations represented by language and performed in order to explore the properties of the model and its application. In fact, the conclusion that all heavy bodies have a unique centre of gravity with certain properties directly results from a reflection on the use of the equilibrium model and the balance-lever model for

[20] The barycentric theory reconstructed by Dijksterhuis (1956, 298-300) found the center by dividing a plane figure into two parts of equal area or weight. However, depending on how the area, and thus the weight, is distributed, the body may or may not in fact be in equilibrium at this point. Asymmetric figures need not balance on the orthogonal plane dividing them in half.

interpreting mechanical devices, as it is illustrated in the treatise by Aristotle and in practical procedures described in Heron.[21] Insofar as these interpretations work, every body to which they apply has a point that corresponds to the fulcrum of a balance. If the identification of this point turns out to be independent of the specific way in which the model is applied to a particular body, it must be unique. Such a conclusion obviously proposes that the application of the model itself has become the object of reflection, justifying our characterization of it as a secondary reflection. Without the representation in terms of language, this could hardly have happened.

According to our interpretation, a similar process of reflective abstraction has brought about the concept of force of heaviness. As we have pointed out, this concept resulted from a reflection on the implication of the balance-lever model that weight differences can be compensated by differences in length. The fact that a small weight may balance a larger weight if it is placed at a larger distance can be interpreted according to the equilibrium model as an equivalence between the force and the weight involved. This interpretation may now trigger a modification of the force concept which is differentiated so that force no longer simply equals the weight but is qualified according to the position of the weight. This can also be found in Vitruvius in a more practical context where he explains how the small counterweight of the steelyard can balance out a “greater force” by its *momentum ponderis*.[22]

On this background we recognize the ancient proofs of the law of the lever as a third step in the genesis of mechanics, after the invention of balances with unequal arms and after the theoretical constitution of the equilibrium and balance-lever models in the Aristotelian mechanics. Although all three steps are temporally close, they are genetically distinct because each step builds on the preceding one. The proofs thus presuppose not only the practical knowledge about balances with equal and unequal arms, which gave rise to the equilibrium and the balance-lever model, but also the constitution of concepts of theoretical knowledge such as centre of gravity and force of heaviness, which result as reflective abstractions from these models. Both proofs involve, however, not only these abstract concepts but also, just like the proofs of Euclidean geometry after which they are modelled, complex constructions corresponding to physical arrangements and the operations performed on them. While in Euclidean geometry the physical operations are constructions performed with compass and ruler, they here correspond to operations with balances. And just as the admissible operations in Euclid’s geometry are formulated in the postulates, the postulates here circumscribe the admissible operations with a balance. In this way, the practical knowledge about balances continued to provide the empirical grounding that made these proofs convincing.

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[21] Heron, *Mechanics*, Nix/Schmidt 1900, 64-67.
[22] Vitruvius, *De architectura*, X.iii.4

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